المقدار	الحل
$\int \sin^2 3x \ \cos 3x \ dx$	$\frac{1}{3}\int \sin^2 3x \ (3\cos 3x)dx =$
$\int \sin^2 3x \ dx$	$\frac{1}{2}\int (1-\cos 6x) \ dx =$
$\int \sin^2 3x \cos^2 3x \ dx$	$\int \frac{1}{2}(1-\cos 6x)\cdot\frac{1}{2}(1+\cos 6x) \ dx = \frac{1}{8}\int \left(1-\cos^2 6x\right)dx$ $=\frac{1}{8}\int \sin^2 6x \, dx = \frac{1}{8}\cdot\frac{1}{2}\int (1-\cos 12x)dx =$
$\int \sin 3x \cos 3x \ dx$	$\frac{1}{3}\int (\sin 3x)^1 (3\cos 3x) \ dx = \frac{1}{6}\sin^2 3x + c$
$\int \cos^4 3x \ dx$	$\int \left(\cos^2 3x\right)^2 \ dx = \int \left(\frac{1}{2}(1+\cos 6x)\right)^2 \ dx =$ $\frac{1}{4}\int (1+\cos 6x)^2 \, dx = \frac{1}{4}\int (1+2\cos 6x + \cos^2 6x)dx =$
$\int \sin^3 3x \ \cos 3x \ dx$	$\frac{1}{3}\int \sin^3 3x \ (3\cos 3x)dx =$
$\int \sin^3 3x \ dx$	$\int \sin 3x \cdot \sin^2 3x \ dx = \int \sin 3x \cdot (1-\cos^2 3x) \ dx$ $= \int \sin 3x \cdot dx + \int \sin 3x (\cos 3x)^2 \ dx$ $= \int \sin 3x \cdot dx - \frac{1}{3}\int \ (\cos 3x)^2 \ (\underline{-3\sin 3x} \ dx) =$
$\int \tan^2 4x \ dx$	$\int \left(\sec^2 4x - 1\right) \ dx =$
$\int \tan^4 4x \ dx$	$\int \tan^2 4x \cdot \tan^2 4x \ dx = \int \tan^2 4x \cdot \left(\sec^2 4x - 1\right)dx$ $=\frac{1}{4}\int (\tan 4x)^2 \cdot \left(4\sec^2 4x\right)dx - \int \left(\sec^2 4x - 1\right)dx =$
$\int \sec^4 4x \ dx$	$\int \sec^2 4x \cdot \left(\sec^2 4x\right)dx = \int \sec^2 4x \cdot \left(\tan^2 4x + 1\right)dx$ $=\frac{1}{4}\int (\tan 4x)^2 \cdot \left(4\sec^2 4x\right)dx + \int \sec^2 4x \ dx =$
$\int \sec^n 4x \ \tan 4x \, dx : n \neq -1$	$\frac{1}{4}\int \sec^{n-1} 4x \cdot \left(4\sec^1 4x \tan 4x\right)dx = \frac{1}{4n}\sec^n + c$
$\int \sqrt{1-\sin 6x} \ dx$	$\int \sqrt{1-2\sin 3x \ \cos 3x} \ dx = \int \sqrt{\cos^2 3x - 2\sin 3x \ \cos 3x + \sin^2 3x} \ dx$ $=\int \sqrt{(\cos 3x - \sin 3x)^2} \ dx = \int \lvert \cos 3x - \sin 3x \rvert dx = \pm(\qquad) + c$
$\int \sqrt{1-\cos 6x} \ dx$	$\int \sqrt{2\sin^2 3x} \ dx = \sqrt{2}\int \lvert \sin 3x \rvert dx = \pm\frac{\sqrt{2}}{3}\cos 3x + c$

المقدار	الحل
$\int \frac{\sin^3 x}{1-\cos x}\,dx$	$\int \frac{\sin^2 x\,\sin x}{1-\cos x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{1-\cos x}\,dx =$ $\int \frac{\cancel{(1-\cos x)}\,(1+\cos x)\sin x}{\cancel{1-\cos x}}\,dx = \int dx + \int (\sin x)^1 \cos x\,dx =$
$\int \frac{2\sin\sqrt[3]{x}}{5\sqrt[3]{x^2}}\,dx$	$\frac{2}{5}\cdot 3\int \sin\sqrt[3]{x}\cdot\frac{1}{3\sqrt[3]{x^2}}\,dx = -\frac{6}{5}\cos\sqrt[3]{x} + c$
$\int \sin 6x\,\cos 3x\,dx$	$\int 2\sin 3x\,\cos 3x\cdot\cos 3x\,dx = 2\int \sin 3x\,(\cos 3x)^2\,dx$ $= -\frac{2}{3}\int (\cos 3x)^2\,(-3\sin 3x)\,dx = -\frac{2}{9}\cos^3 3x + c$
$\int \cos 6x\,\cos 3x\,dx$	$\int \left(1-2\sin^2 3x\right)\cos 3x\,dx = \int \cos 3x\,dx - 2\int (\sin 3x)^2 \cos 3x\,dx =$
$\int \cos 6x\,\sin 3x\,dx$	$\int \left(2\cos^2 3x - 1\right)\sin 3x\,dx = 2\int (\cos 3x)^2 \sin 3x\,dx - \int \sin 3x\,dx =$
$\int \frac{\cos^2 6x}{\sin 3x - \cos 3x}\,dx$	$\int \frac{\cos^2 3x - \sin^2 3x}{\cos 3x - \sin 3x}\,dx = \int \frac{\cancel{(\cos 3x - \sin 3x)}\,(\cos 3x + \sin 3x)}{\cancel{\cos 3x - \sin 3x}}\,dx =$
$\int \left(\frac{\sin x}{\sqrt{x}} + 2\sqrt{x}\,\cos x\right)dx$	$2\int \left(\frac{\sin x}{2\sqrt{x}} + \sqrt{x}\,\cos x\right)dx = 2\int \frac{d}{dx}\left(\sqrt{x}\,\sin x\right)dx = 2\sqrt{x}\,\sin x + c$
$\int \tan 4x\,dx$	$\int \frac{\sin 4x}{\cos 4x}\,dx = \frac{-1}{4}\int \frac{1}{\cos 4x}\,(-4\sin 4x)\,dx = -\frac{1}{4}\ln\lvert\sin 4x\rvert + c$
$\int \sec x\,\sin x\,dx$	$-\int \frac{1}{\cos x}(-\sin x)\,dx = -\ln\lvert\cos x\rvert + c = \ln\lvert\sec x\rvert + c$
$\int \sec x\,dx$	$\int \sec x\cdot\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx = \int \frac{1}{\sec x + \tan x}\cdot(\sec x + \tan x)\,dx$ $= \ln\lvert\sec x + \tan x\rvert + c$
$\int \sec^2 4x\; e^{\tan 4x}\,dx$	$\frac{1}{4}\int e^{\tan 4x}\left(4\sec^2 4x\right)dx = \frac{1}{4}e^{\tan 4x} + c$
$\int \sin 6x\,\cos 6x\;3^{\cos 12x}\,dx$	$\int \sin 6x\,\cos 6x\;3^{\cos 12x}\,dx = \frac{1}{2}\int 3^{\cos 12x}\cdot\sin 12x\,dx$ $= -\frac{1}{24}\int 3^{\cos 12x}\cdot(-12\sin 12x)\,dx = -\frac{1}{24\ln 3}\cdot 3^{\cos 12x} + c$
$\int e^{-2\ln(\cos x)}\,dx$	$\int e^{\ln(\cos^{-2} x)}\,dx = \int \cos^{-2} x\,dx = \int \sec^2 x\,dx = \tan x + c$